

**Review: rational functions (with solutions)**

1. Sketch the graph of the following rational functions (remember the “5 steps”)

a.  $R(x) = \frac{x+1}{x(x+4)}$     b.  $R(x) = \frac{3x+3}{2x+4}$     c.  $R(x) = \frac{3}{x^2-4}$

d.  $f(x) = \frac{x^4+x^2+1}{x^2-1}$     e.  $f(x) = \frac{x^3-1}{x^2-9}$     f.  $f(x) = \frac{x^2}{x^2+x-6}$

g.  $f(x) = \frac{x}{x^2-4}$     h.  $f(x) = \frac{3}{(x-1)(x^2-4)}$     i.  $f(x) = \frac{4(x^2-1)}{x^2-16}$

j.  $f(x) = \frac{x^2-3x-4}{x+2}$     k.  $f(x) = \frac{x^2+x-12}{x-4}$     l.  $f(x) = \frac{x^2+x-12}{x+2}$

m.  $f(x) = \frac{x(x-1)^2}{(x+3)^3}$     n.  $f(x) = \frac{x^2+x-12}{x^2-x-6}$

**(Remark:** ignore the “oblique asymptotes” questions)

Solutions below.

#### 4.4 Concepts and Vocabulary (page 353)

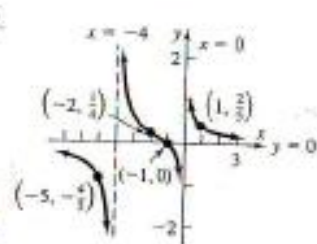
3. in lowest terms 4. False 5. False 6. True

#### 4.4 Exercises (page 353)

1. Domain:  $\{x|x \neq 0, x \neq -4\}$
2.  $x$ -intercept:  $-1$ ; no  $y$ -intercept
3. No  $y$ -axis or origin symmetry
4. Vertical asymptotes:  $x = 0, x = -4$
5. Horizontal asymptote:  $y = 0$ , intersected at  $(-1, 0)$
- 6.

$$R(x) = \frac{x+1}{x(x+4)}$$

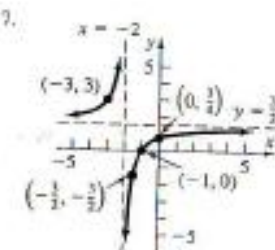
	-4	-1	0
Interval	$(-\infty, -4)$	$(-4, -1)$	$(-1, 0)$
Number Chosen	-5	-2	$-\frac{1}{2}$
Value of $R$	$R(-5) = -\frac{4}{5}$	$R(-2) = \frac{1}{4}$	$R(-\frac{1}{2}) = -\frac{1}{2}$
Location of Graph	Below $x$ -axis	Above $x$ -axis	Below $x$ -axis
Point on Graph	$(-5, -\frac{4}{5})$	$(-2, \frac{1}{4})$	$(-\frac{1}{2}, -\frac{1}{2})$



1. Domain:  $\{x|x \neq -2\}$
2.  $x$ -intercept:  $-1$ ;  $y$ -intercept:  $\frac{3}{4}$
3. No  $y$ -axis or origin symmetry
4. Vertical asymptote:  $x = -2$
5. Horizontal asymptote:  $y = \frac{3}{2}$ , not intersected
- 6.

$$R(x) = \frac{3x+3}{2x+4}$$

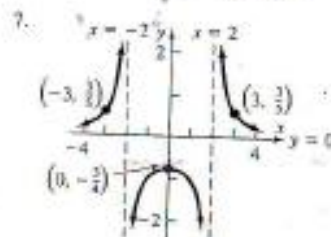
	-2	-1
Interval	$(-\infty, -2)$	$(-2, -1)$
Number Chosen	-3	$-\frac{3}{2}$
Value of $R$	$R(-3) = 3$	$R(-\frac{3}{2}) = -\frac{3}{2}$
Location of Graph	Above $x$ -axis	Below $x$ -axis
Point on Graph	$(-3, 3)$	$(-\frac{3}{2}, -\frac{3}{2})$



1. Domain:  $\{x|x \neq -2, x \neq 2\}$
2. No  $x$ -intercept;  $y$ -intercept:  $-\frac{3}{4}$
3. Symmetric with respect to  $y$ -axis
4. Vertical asymptotes:  $x = 2, x = -2$
5. Horizontal asymptote:  $y = 0$ , not intersected
- 6.

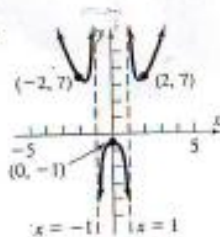
$$R(x) = \frac{3}{x^2-4}$$

	-2	2
Interval	$(-\infty, -2)$	$(-2, 2)$
Number Chosen	-3	0
Value of $R$	$R(-3) = \frac{3}{5}$	$R(0) = -\frac{3}{4}$
Location of Graph	Above $x$ -axis	Below $x$ -axis
Point on Graph	$(-3, \frac{3}{5})$	$(0, -\frac{3}{4})$



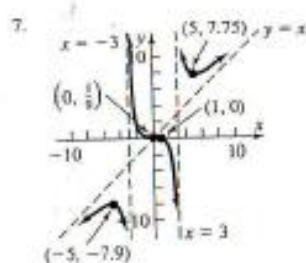
3. 1. Domain:  $\{x|x \neq -1, x \neq 1\}$   $f(x) = \frac{x^4 + x^2 + 1}{x^2 - 1}$   
 2. No  $x$ -intercept;  $y$ -intercept:  $-1$   
 3. Symmetric with respect to  $y$ -axis  
 4. Vertical asymptotes:  $x = -1, x = 1$   
 5. No horizontal or oblique asymptotes  
 6.

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Number Chosen	$-2$	$0$	$2$
Value of $P$	$P(-2) = 7$	$P(0) = -1$	$P(2) = 7$
Location of Graph	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-2, 7)$	$(0, -1)$	$(2, 7)$



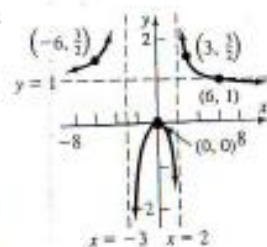
15. 1. Domain:  $\{x|x \neq -3, x \neq 3\}$   $f(x) = \frac{x^3 - 1}{x^2 - 9}$   
 2.  $x$ -intercept:  $1$ ;  $y$ -intercept:  $\frac{1}{9}$   
 3. No  $y$ -axis or origin symmetry  
 4. Vertical asymptotes:  $x = 3, x = -3$   
 5. Oblique asymptote:  $y = x$ , intersected at  $(\frac{1}{9}, \frac{1}{9})$   
 6.

	$(-\infty, -3)$	$(-3, 1)$	$(1, 3)$	$(3, \infty)$
Number Chosen	$-4$	$0$	$2$	$4$
Value of $H$	$H(-4) = -9.3$	$H(0) = \frac{1}{9}$	$H(2) = -1.4$	$H(4) = 9$
Location of Graph	Below $x$ -axis	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-4, -9.3)$	$(0, \frac{1}{9})$	$(2, -1.4)$	$(4, 9)$



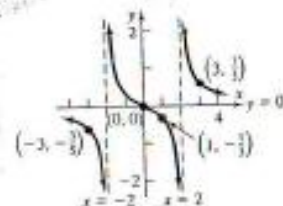
17. 1. Domain:  $\{x|x \neq -3, x \neq 3\}$   $f(x) = \frac{x^2}{x^2 + x - 6}$   
 2.  $x$ -intercept:  $0$ ;  $y$ -intercept:  $0$   
 3. No  $y$ -axis or origin symmetry  
 4. Vertical asymptotes:  $x = 2, x = -3$   
 5. Horizontal asymptote:  $y = 1$ , intersected at  $(6, 1)$   
 6.

	$(-\infty, -3)$	$(-3, 0)$	$(0, 2)$	$(2, \infty)$
Number Chosen	$-6$	$-1$	$1$	$3$
Value of $R$	$R(-6) = 1.5$	$R(-1) = -\frac{1}{6}$	$R(1) = -0.25$	$R(3) = 1.5$
Location of Graph	Above $x$ -axis	Below $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-6, 1.5)$	$(-1, -\frac{1}{6})$	$(1, -0.25)$	$(3, 1.5)$



19. 1. Domain:  $\{x|x \neq -2, x \neq 2\}$   $f(x) = \frac{x}{x^2 - 4}$   
 2.  $x$ -intercept:  $0$ ;  $y$ -intercept:  $0$   
 3. Symmetry with respect to origin  
 4. Vertical asymptotes:  $x = -2, x = 2$   
 5. Horizontal asymptote:  $y = 0$ , intersected at  $(0, 0)$   
 6.

	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
Number Chosen	$-3$	$-1$	$1$	$3$
Value of $G$	$G(-3) = -\frac{3}{5}$	$G(-1) = \frac{1}{3}$	$G(1) = -\frac{1}{3}$	$G(3) = \frac{3}{5}$
Location of Graph	Below $x$ -axis	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-3, -\frac{3}{5})$	$(-1, \frac{1}{3})$	$(1, -\frac{1}{3})$	$(3, \frac{3}{5})$

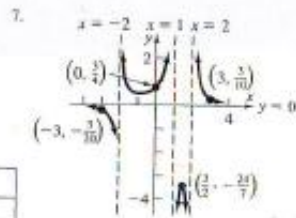


21. 1. Domain:  $\{x|x \neq 1, x \neq -2, x \neq 2\}$

$$f(x) = \frac{3}{(x-1)(x^2-4)}$$

2. No  $x$ -intercept;  $y$ -intercept:  $\frac{3}{4}$   
 3. No  $y$ -axis or origin symmetry  
 4. Vertical asymptotes:  $x = -2, x = 1, x = 2$   
 5. Horizontal asymptote:  $y = 0$ , not intersected

	$\xrightarrow{\hspace{10em}}$			
	-2	1	2	
Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, 2)$	$(2, \infty)$
Number Chosen	-3	0	1.5	3
Value of $R$	$R(-3) = -\frac{3}{20}$	$R(0) = \frac{3}{4}$	$R(1.5) = -\frac{24}{7}$	$R(3) = \frac{3}{10}$
Location of Graph	Below $x$ -axis	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-3, -\frac{3}{20})$	$(0, \frac{3}{4})$	$(1.5, -\frac{24}{7})$	$(3, \frac{3}{10})$

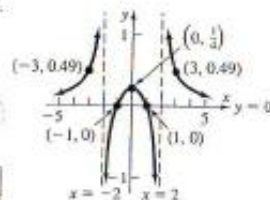


23. 1. Domain:  $\{x|x \neq -2, x \neq 2\}$

$$f(x) = \frac{4(x^2-1)}{x^4-16}$$

2.  $x$ -intercepts: -1, 1;  $y$ -intercept:  $\frac{1}{4}$   
 3. Symmetry with respect to  $y$ -axis  
 4. Vertical asymptotes:  $x = -2, x = 2$   
 5. Horizontal asymptote:  $y = 0$ , intersected at  $(-1, 0)$  and  $(1, 0)$

	$\xrightarrow{\hspace{10em}}$				
	-2	-1	1	2	
Interval	$(-\infty, -2)$	$(-2, -1)$	$(-1, 1)$	$(1, 2)$	$(2, \infty)$
Number Chosen	-3	-1.5	0	1.5	3
Value of $H$	$H(-3) = 0.49$	$H(-1.5) = -0.46$	$H(0) = \frac{1}{4}$	$H(1.5) = -0.46$	$H(3) = 0.49$
Location of Graph	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-3, 0.49)$	$(-1.5, -0.46)$	$(0, \frac{1}{4})$	$(1.5, -0.46)$	$(3, 0.49)$

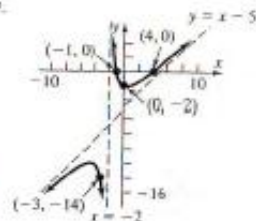


25. 1. Domain:  $\{x|x \neq -2\}$

$$f(x) = \frac{x^2-3x-4}{x+2}$$

2.  $x$ -intercepts: -1, 4;  $y$ -intercept: -2  
 3. No  $y$ -axis or origin symmetry  
 4. Vertical asymptote:  $x = -2$   
 5. Oblique asymptote:  $y = x - 5$ , not intersected

	$\xrightarrow{\hspace{10em}}$			
	-2	-1	4	
Interval	$(-\infty, -2)$	$(-2, -1)$	$(-1, 4)$	$(4, \infty)$
Number Chosen	-3	-1.5	0	5
Value of $F$	$F(-3) = -14$	$F(-1.5) = 5.5$	$F(0) = -2$	$F(5) = 0.86$
Location of Graph	Below $x$ -axis	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-3, -14)$	$(-1.5, 5.5)$	$(0, -2)$	$(5, 0.86)$

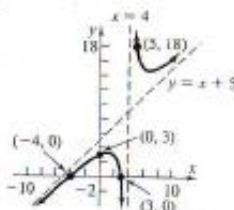


27. 1. Domain:  $\{x|x \neq 4\}$

$$f(x) = \frac{x^2+x-12}{x-4}$$

2.  $x$ -intercepts: -4, 3;  $y$ -intercept: 3  
 3. No  $y$ -axis or origin symmetry  
 4. Vertical asymptote:  $x = 4$   
 5. Oblique asymptote:  $y = x + 5$ , not intersected

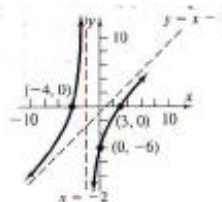
	$\xrightarrow{\hspace{10em}}$			
	-4	3	4	
Interval	$(-\infty, -4)$	$(-4, 3)$	$(3, 4)$	$(4, \infty)$
Number Chosen	-5	0	3.5	5
Value of $R$	$R(-5) = -\frac{8}{9}$	$R(0) = 3$	$R(3.5) = -7.5$	$R(5) = 18$
Location of Graph	Below $x$ -axis	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-5, -\frac{8}{9})$	$(0, 3)$	$(3.5, -7.5)$	$(5, 18)$



$$f(x) = \frac{x^2 + x - 12}{x + 2}$$

29. 1. Domain:  $\{x | x \neq -2\}$   
 2.  $x$ -intercepts:  $-4, 3$ ;  $y$ -intercept:  $-6$   
 3. No  $y$ -axis or origin symmetry  
 4. Vertical asymptote:  $x = -2$   
 5. Oblique asymptote:  $y = x - 1$ , not intersected  
 6.

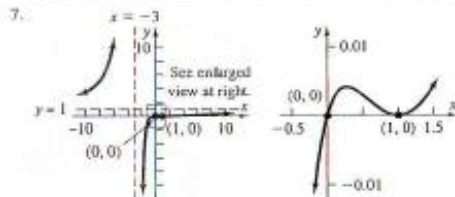
Interval	$(-\infty, -4)$	$(-4, -2)$	$(-2, 3)$	$(3, \infty)$
Number Chosen	$-5$	$-3$	$0$	$4$
Value of $F$	$F(-5) = -\frac{3}{2}$	$F(-3) = 6$	$F(0) = -6$	$F(4) = \frac{4}{3}$
Location of Graph	Below $x$ -axis	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis
Point on Graph	$(-5, -\frac{3}{2})$	$(-3, 6)$	$(0, -6)$	$(4, \frac{4}{3})$



31. 1. Domain:  $\{x | x \neq -3\}$   
 2.  $x$ -intercepts:  $0, 1$ ;  $y$ -intercept:  $0$   
 3. No  $y$ -axis or origin symmetry  
 4. Vertical asymptote:  $x = -3$   
 5. Horizontal asymptote:  $y = 1$ , not intersected  
 6.

$$f(x) = \frac{x(x-1)^2}{(x+3)^3}$$

Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 1)$	$(1, \infty)$
Number Chosen	$-4$	$-1$	$\frac{1}{2}$	$2$
Value of $R$	$R(-4) = 100$	$R(-1) = -0.5$	$R(\frac{1}{2}) \approx 0.003$	$R(2) = 0.016$
Location of Graph	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis	Above $x$ -axis
Point on Graph	$(-4, 100)$	$(-1, -0.5)$	$(\frac{1}{2}, 0.003)$	$(2, 0.016)$



33. 1. Domain:  $\{x | x \neq -2, x \neq 3\}$   
 2.  $x$ -intercept:  $-4$ ;  $y$ -intercept:  $2$   
 3. No  $y$ -axis or origin symmetry  
 4. Vertical asymptote:  $x = -2$ ; hole at  $(3, \frac{7}{5})$   
 5. Horizontal asymptote:  $y = 1$ , not intersected  
 6.

$$f(x) = \frac{x^2 + x - 12}{x^2 - x - 6}$$

Interval	$(-\infty, -4)$	$(-4, -2)$	$(-2, 3)$	$(3, \infty)$
Number Chosen	$-5$	$-3$	$0$	$4$
Value of $R$	$R(-5) = \frac{1}{3}$	$R(-3) = -1$	$R(0) = 2$	$R(4) = \frac{4}{3}$
Location of Graph	Above $x$ -axis	Below $x$ -axis	Above $x$ -axis	Above $x$ -axis
Point on Graph	$(-5, \frac{1}{3})$	$(-3, -1)$	$(0, 2)$	$(4, \frac{4}{3})$

